Alleviating Non-identifiability: A High-Fidelity Calibration Objective for Financial Market Simulation With Multivariate Time Series Data

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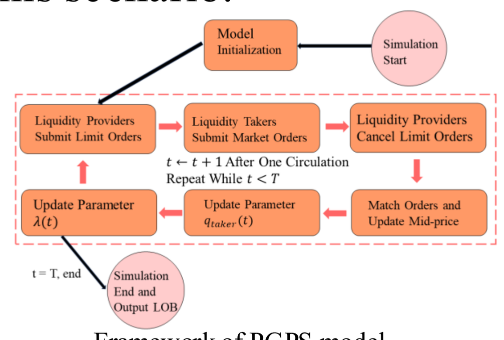
I. Background

Agent-based Modeling (ABM) & Financial Market Simulation (FMS)

- ABM simulates complex systems by modeling multiple types of agents and their interactions.
- FMS reproduces market dynamics from the Limit Order Book (LOB), generating multi-variate time series such as prices, volumes, and spreads.

Black-box Optimization for Calibration

- **Calibration** adjusts simulation parameters so that simulated outputs closely match real data. The objective is **non-differentiable and computationally expensive.**
- **Evolutionary algorithms** are particularly effective for finding optimal parameters in this scenario.



Framework of PGPS model

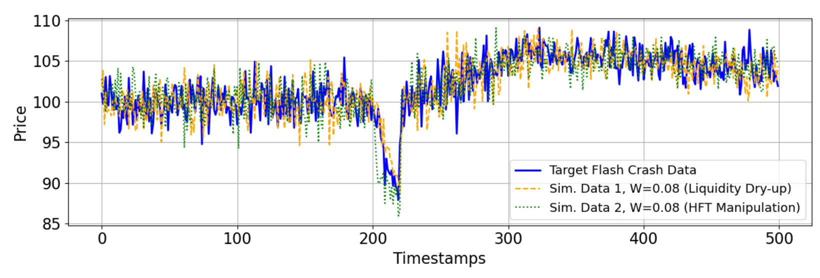
Framework of FMS

II. Motivation

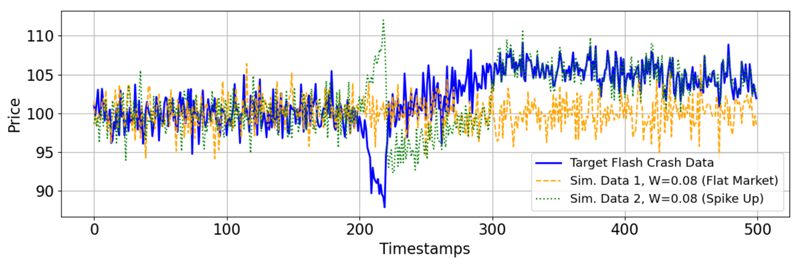
Previous Methods

Match output on a single feature, typically mid-price, causing the **Non-identifiability Problem (NIP)**, i.e., different parameters yield the same discrepancy values yet different market mechanisms.

Severity of NIP



Scenario1: Similar trend, different mechanism



Scenario2: Same discrepancy, different trend

Our Method

Use multiple distinct features in calibration to filter out parameters that resemble the target data on one feature but deviate from others, thereby alleviating NIP.

III. Theoretical Guarantee

Setup and Univariate Non-identifiability (NI)

- Calibration Task: $D\left(\widehat{X_T^k}, M(\omega) = X_T^k\right)$, $\widehat{X_T^k}, X_T^k$ are target and simulated data of feature k, $M(\omega)$ is the agent-based model, here $\omega = [\alpha, \mu, \delta, \Delta_s, \lambda_0, C_\lambda]$.
- Non-identifiable Set: $S^{D,k,\epsilon} \subseteq$ parameter space $\Omega^{D,k}$, s.t. $\forall\, \omega_1, \omega_2 \in S^{D,k,\epsilon}$,
$$D\left(\widehat{X_T^k}, M(\omega_1)\right) < \epsilon, D\left(\widehat{X_T^k}, M(\omega_2)\right) < \epsilon.$$
- Univariate NI: $P\left(\omega \in S^{D,k,\epsilon} \middle| \widehat{X_T^k}\right) = \frac{\mu(S^{D,k,\epsilon})}{\mu(\Omega^{D,k})}$, μ denotes Lebesgue Measure.

Multivariate NI

- Assume $\Omega^{D,k} = \Omega^D$ for all k, and the total number of features is K.
- Multivariate NI: $P(\omega \in \cap_{k=1}^K S^{D,k,\epsilon} \,|\widehat{X_T}) = \frac{\mu(\cap_{k=1}^K S^{D,k,\epsilon})}{\mu(\Omega^D)}, \widehat{X_T} = \left\{\widehat{X_T^k}\right\}_{k=1}^K$.
- Define the overlap ratio: $\beta_1 = \frac{\mu(S^{D,1,\epsilon})}{\mu(\Omega^D)}, \beta_i = \frac{\mu(\cap_{j=1}^i S^{D,k,\epsilon})}{\mu(\cap_{j=1}^{i-1} S^{D,k,\epsilon})}, i = 2, \cdots K.$

Theorem 1[The Exponential Alleviation]

If $\beta_i < \frac{\sum_{j=1}^{i-1} {\beta_j}^2}{i-1}$, the upper bound of multivariate non-identifiability is reduced exponentially with the number of features used in calibration, i.e.,

$$P(\omega \in \cap_{k=1}^K S^{D,k,\epsilon} \,|\widehat{X_T}) = \prod_{i=1}^K \beta_i \le \left(\frac{\sum_{i=1}^K {\beta_i}^2}{K}\right)^{\frac{K}{2}} < (1 - \triangle_K)^{\frac{K}{2}} \le e^{-\lambda \cdot K},$$

where $\triangle_K = 1 - \frac{K-1}{K} \cdot \left(1 + \frac{{\beta_K}^2}{\sum_{i=1}^{K-1} {\beta_i}^2}\right), \lambda = \inf_K\left(-\frac{1}{2}\ln(1 - \triangle_K)\right)$

Theorem 2 [The Utility and Uniqueness of F]

Let F aggregate the K individual calibration tasks via **maximization**, i.e.,

$$F(\widehat{X_T}, M(\omega) = X_T) = \max_k D\left(\widehat{X_T^k}, M(\omega) = X_T^k\right)$$

then minimizing F **uniquely** achieves $\cap_{k=1}^K S^{D,k,\epsilon} = S^{F,\epsilon}$.

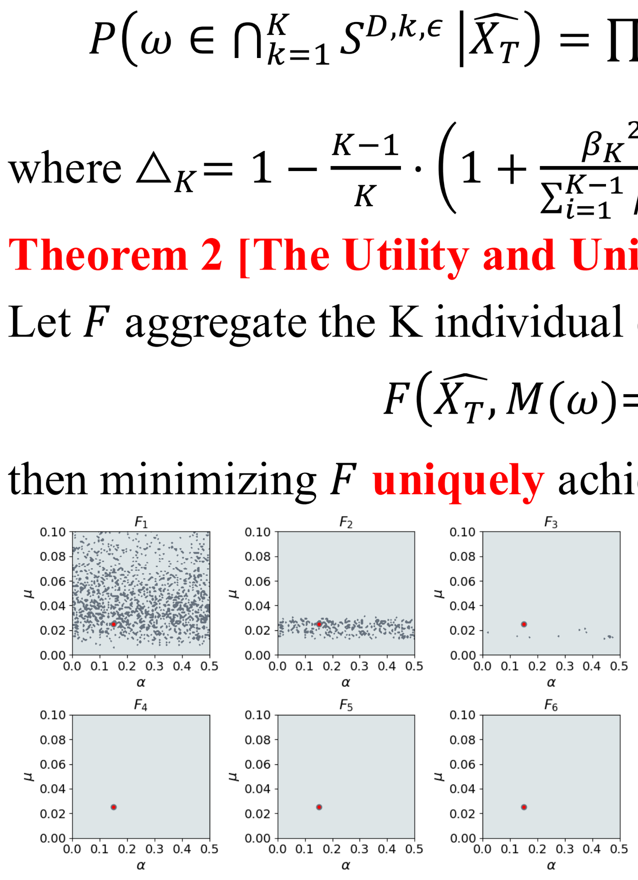


Illustration of Theorem 1: Exponential reduction of the non-identifiable set (grey dots) with increasing features; the red dot denotes the optimum

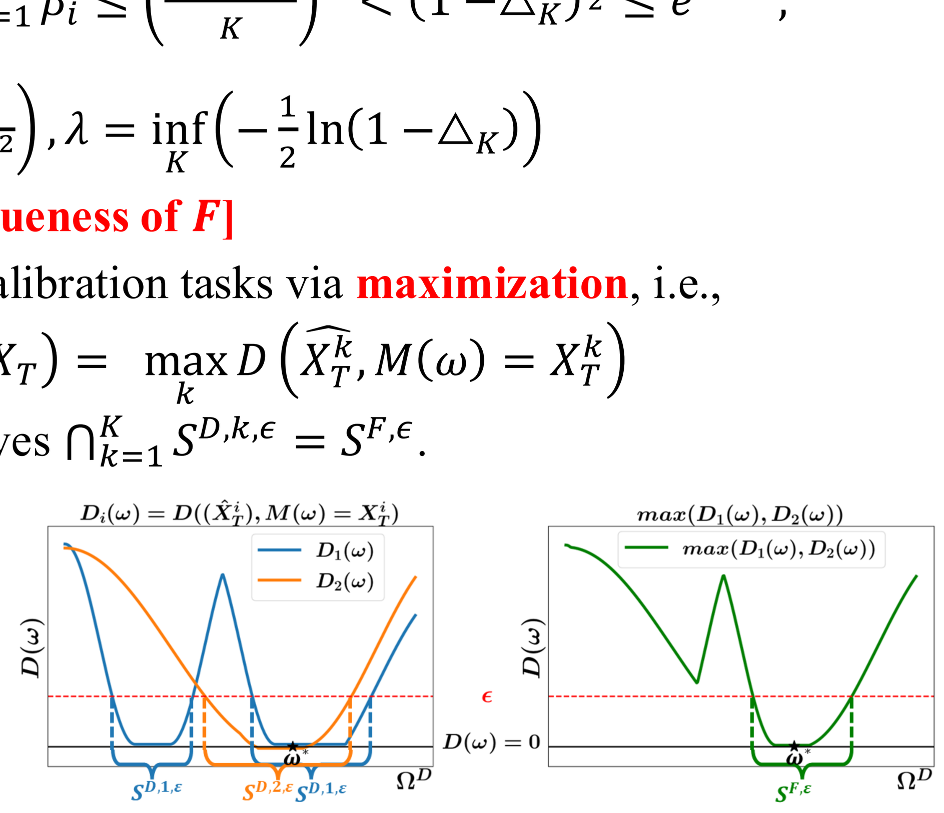
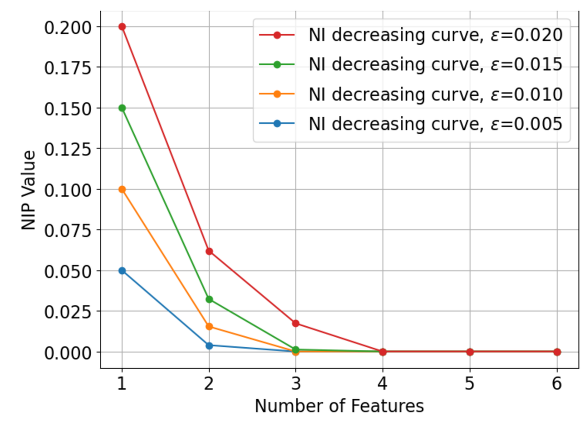


Illustration of Theorem 2: Optimizing the maximization of two functions equals searching in their intersections

IV. Experiments

Effectiveness of Multiple Features

Multi-feature calibration significantly reduces NI values, showing exponential alleviation of non-identifiability.



NI decreases exponentially with the number of features

Six commonly used features are selected:

- *f_1, the mid-price at each t-th step;*
- *f_2, the total traded volume within each t-th step;*
- *f_3, the price return at each t-th step;*
- *f_4, the spread at each t-th step;*
- *f_5, the volume of the best bid price at each t-th step;*
- *f_6, the volume of the best ask price at each t-th step.*

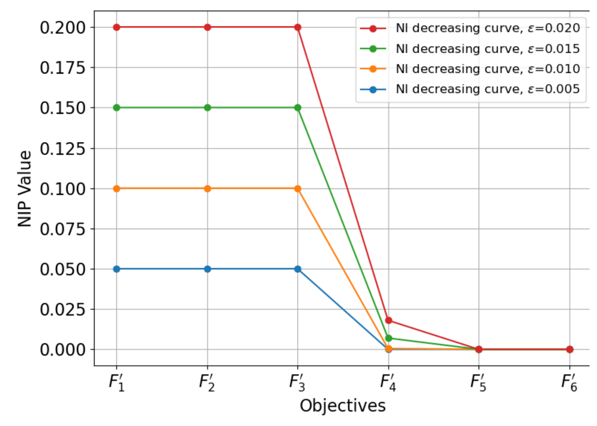
$F_1 \sim F_6$: Calibration objective formed by sequentially adding $f_1 \sim f_6$,

$$F_i = \max_{k \le i} D\left(\widehat{X_T^k}, M(\omega)\right)$$

Optimization method: Particle Swarm Optimization (PSO)

Properties of the Selected Features

Selected features are appropriate, maximizing their contribution to NI reduction.



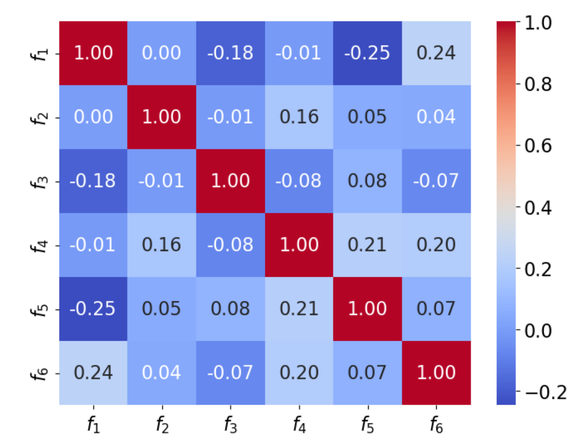
Impact of feature dependencies on NI reduction

Six new features are derived from the originals:

- $f_1' = f_1, f_2' = 2f_1$;
- $f_3' = -2f_1, f_4' = f_5$;
- $f_5' = f_6, f_6' = f_2$.

Objectives F_i' are built sequentially as:

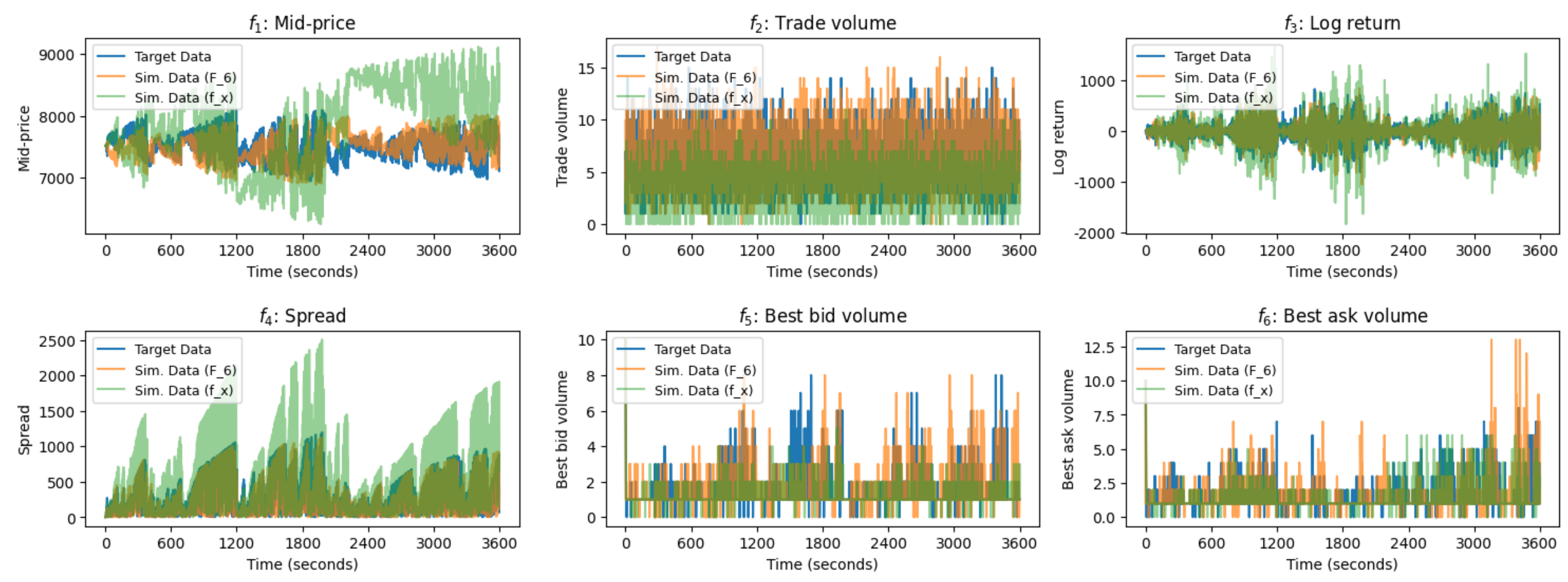
$$F_i' = \max_{k \le i} D\left(\widehat{X_T^k}, M(\omega)\right).$$



Pearson correlation of the selected features

Calibration to the Synthetic Data

The proposed objective achieves significant superior simulation fidelity.



Simulated vs. target series for each selected feature, comparing calibration with F_6 and the best single-feature objective f_x

Statistical Ablation Study of Aggregation Methods and Distance Measure

Max-based aggregation with Wasserstein distance consistently outperforms others.

	f_1	f_2	f_3	f_4	f_5	f_6	F_6
Data 1	3.99	5.07	6.22	4.62	4.92	3.95	**3.07**
Data 2	3.65	3.95	4.67	3.73	6.18	4.77	**2.90**
Data 3	**2.86**	4.43	3.73	3.87	5.75	4.62	3.08
Data 4	2.83	3.50	3.47	3.87	4.33	2.99	**2.47**
Data 5	2.72	**2.70**	4.93	5.98	3.17	5.83	2.84
Data 6	5.23	5.58	4.02	6.67	4.48	6.77	**3.67**
Data 7	3.08	3.57	**3.06**	3.65	3.60	3.86	3.51
Data 8	4.88	4.50	2.56	3.82	3.91	3.62	**2.26**
Data 9	4.31	3.59	5.07	4.19	5.73	3.82	**3.51**
Data 10	2.53	2.43	2.95	6.93	3.22	2.24	**2.23**
#rank	3.20^+	4.10^+	4.00^+	5.10^+	5.60^+	4.50^+	**1.50**

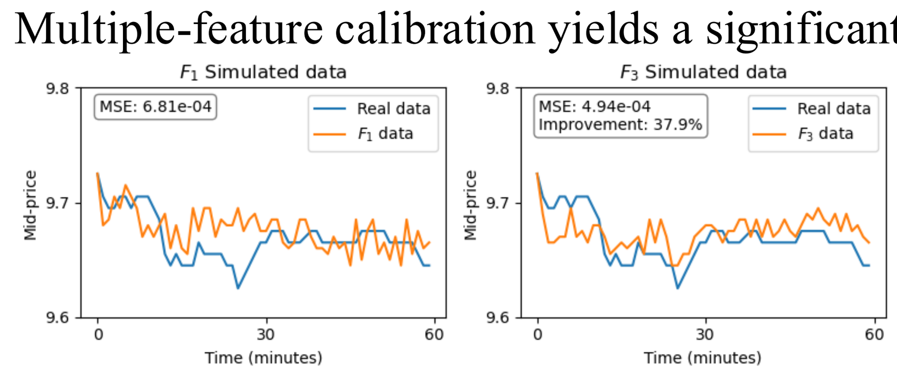
Table I: Comparisons of calibration performance between 6 univariate data and F_6.

aggregate	max				min	mean
measure	KL	KS	MSE	W	W	W
data 1	3.19	4.47	3.47	**3.07**	6.63	4.77
data 2	6.03	3.08	3.03	**2.90**	3.36	3.89
data 3	6.86	4.30	3.19	**3.08**	4.63	3.47
data 4	3.01	3.10	3.90	**2.47**	4.32	4.33
data 5	**2.55**	6.58	3.75	2.84	3.01	3.79
data 6	3.71	6.69	4.76	**3.67**	3.74	5.73
data 7	3.93	3.10	**1.42**	3.51	3.23	3.52
data 8	3.82	3.18	4.65	**2.26**	3.88	2.54
data 9	8.35	5.48	3.85	**3.51**	5.07	5.40
data 10	3.69	3.63	**2.10**	2.23	5.32	4.04
#rank	3.90^+	3.90^+	2.90^-	**1.50**	4.30^+	4.50^+

Table II: Statistical ablation study of aggregation methods and distance measure

Calibration to the Real Data

Multiple-feature calibration yields a significant higher mid-price fidelity (37.9%).



Comparison of mid-price series between real data and simulated data under single-feature F_1 and multi-feature F_3 calibration.

The real data is chosen as 000001.SZ from Shenzhen Stock Exchange of China, consisting of 1200 time steps at a frequency of 3 seconds from 9:30 a.m. to 10:30 a.m. of a day in 2019.

V. Summary

Theory:

Non-identifiability in financial market simulation is formally defined, and theoretical analysis demonstrates that aggregating appropriate features in calibration process can exponentially alleviate this problem.

A max-based calibration objective is introduced, uniquely preserving the intersection property of non-identifiability sets.

Experiments:

Both synthetic and real market data experiments show significant improvement in simulation fidelity with the max-based calibration objective.

Contact & Publication

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