## SUSTech Department of Statistics and Data Science

#### MA204: Mathematical Statistics (数理统计)

(2023 Spring Semester)

Lecturer: Prof. Gary Guo-Liang TIAN (田国梁) SBB-F3 Room 332

Tutor: Mr. Chenkai WANG (王晨凯) Email: 12232885@mail.sustech.edu.cn

Miss Yuehua GAN (甘月华) Email: 12232872@mail.sustech.edu.cn Mr. Runsong ZHANG (张润松) Email: 12232881@mail.sustech.edu.cn

Note: School of Business Building, The 3-rd Floor = SBB-F3 (商学院三楼)

Lecture Hours: Monday (每周 7 – 8 节) 16:20 – 18:10

Thursday (only odd week, 单周 1-2 节) 08:00-09:50

Venue: The Third Teaching Building, Room 204 (三教108)

Number of QQ: 722 543 392 (授课课件QQ群号码)

#### 1. Course Objectives (课程目标)

This course is on the basis of "MA215: Probability" or "MA212: Probability and Statistics" It will further study the concepts and methods of statistics. The course will lay emphasis on the estimation and hypothesis testing, the two major areas of statistical inference. Through the study of this course, students will be equipped with both quantitative skills and qualitative perceptions essential for making rigorous statistical analysis of data.

本课程是在 MA215 概率论或 MA212 概率论与统计的基础上研究统计概念和统计方法的,它将着重讲述统计推断的两大主题---估计和假设检验。通过学习本课程,学生将具备定性分析,定量分析统计数据的能力。

## 2. <u>Pre-requistes</u> (先修要求)

- MA101a/102a Mathematical Analysis I/II (MA101a/102a 数学分析 I/II)
   「或 MA101b/102b Calculus I/II (MA101b/102/b 高等数学 A(上)/(下)) ]
- MA103b/104b Linear Algebra I/II (MA103b/104b 线性代数上/下)
- MA215 Probability (MA215 概率论)

[或 MA 212 Probability and Statistics (MA 212 概率论与统计)]

## 3. Course Contents (教学内容)

## **Chapter 1. Probability and Distributions**

- 1.1 Probability
- 1.2 Conditional Probability
- 1.3 Bayes Theorem
- 1.4 Probability Distributions
- 1.5 Bivariate Distributions
- 1.6 Expectation, Variance and Moments
- 1.7 Moment Generating Function
- 1.8 Beta and Gamma Distributions
- 1.9 Bivariate Normal Distribution
- 1.10 Inverse Bayes Formulae
- 1.11 Categorical Distribution
- 1.12 Zero-inflated Poisson Distribution

#### **Chapter 2. Sampling Distributions**

- 2.1 Distribution of the Function of Random Variables
- 2.2 Statistics, Sample Mean and Sample Variance
- 2.3 The *t* and *F* Distributions
- 2.4 Order Statistics
- 2.5 Limit Theorems
- 2.6 Some Challenging Questions

## **Chapter 3. Point Estimation**

- 3.1 Maximum Likelihood Estimator (MLE)
- 3.2 Moment Estimator
- 3.3 Bayesian Estimator
- 3.4 Properties of Estimators
- 3.5 Limiting Properties of MLE
- 3.6 Some Challenging Questions

## **Chapter 4. Confidence Interval (CI) Estimation**

- 4.1 Introduction
- 4.2 The CI of Normal Mean
- 4.3 The CI of the Difference of Two Normal Means
- 4.4 The CI of Normal Variance
- 4.5 The CI of the Ratio of Two Normal Variances
- 4.6 Large-Sample Confidence Intervals
- 4.7 The Shortest Confidence Interval

## **Chapter 5. Hypothesis Testing**

- 5.1 Introduction
- 5.2 The Neyman Pearson Lemma
- 5.3 Likelihood Ratio Test
- 5.4 Tests on Normal Means
- 5.5 Goodness of Fit Test

#### Chapter 6. Critical Regions and p-values for Skew Null Distributions

- 6.1 One-sample Chi-square Test on Normal Variance
- 6.2 Two-sample F Test on Normal Variances

**Appendix A: Basic Statistical Distributions** 

**Appendix B: A Unified Expectation Technique** 

Appendix C: The Newton--Raphson and Fisher Scoring Algorithms

4. Learning Objectives and Outcomes (教学目标)

On successful completion of the course, students should be able to:

- understand the importance of sufficient statistic(s) in data reduction and statistical inferences such as point estimation, confidence interval estimation, and testing hypothesis;
- derive maximum likelihood estimators of parameters to calculate maximum likelihood estimates:
- locate pivotal quantity to construct confidence intervals of parameters;
- find testing statistic to test hypotheses associated with one-sample and/or two-sample normal distributions with small sample sizes and non-normal distributions with large sample sizes.

成功完成该课的教学之后,学生可以达到:

- (1) 理解充分统计量在数据压缩和点估计、置信区间估计、假设检验等统计推断中的重要性;
- (2) 导出参数的极大似然估计量并且能计算极大似然估计值;
- (3) 学会通过确定枢轴量来构建参数置信区间;
- (4) 对小样本的正态分布下的单样本或者双样本,能找到假设检验的检验统计量; 对大样本的非正态分布下的单样本或者双样本,能找到假设检验的检验统计量。

### 5. <u>Teaching Hours</u>(上课安排)

The course consists of 48 lectures (3 lectures per week; 16 weeks x = 48 hours), where 12 lectures will be tutorials (12 hours).

#### 6. Assessment (成绩评定)

The assessment consists of five assignments (25%), a 2-hour written midterm test (25%) and a three-hour written examination (50%). Partially or wholly copied assignments will be penalized and/or reported as plagiarism.

## 7. Policy on absence from midterm/class test (期中考试缺考之规定)

If you are or have been unable to attend the mid-term/class test, and if you wish to have a supplementary mid-term/class test, all students should write to the course lecturer giving reasons for your absence, within **7 days** of the absence.

A supplementary midterm/class test is normally granted to those absent from the original test due to illness and with original medical certificate provided. Students absent due to other reasons, in general, are not granted a supplementary midterm/test unless with very special circumstances and with valid documental proofs provided.

Alternatively, students absent from the midterm/class test with a reason might choose the scheme that the weight of the final examination changes from 50% to 75%.

#### 8. Textbook(授课教材)

[1] 田国梁, 蒋学军編著(2021). Mathematical Statistics. 科学出版社, 北京。

## 9. Main References(参考书)

- [2] Miller, I. and Miller, M. (2004). **John E. Freund's Mathematical Statistics with Applications (7th Edition).** Prentice-Hall, New Jersey.
- [3] Hogg, R. V., McKean, J. W. and Craig, A. T. (2014). **Introduction to Mathematical Statistics (7th Edition)**. Pearson Education Limited.
- [4] Arnold, S.F. (1990). **Mathematical Statistics**. Prentice Hall, New Jersey.
- [5] Casella G. and Berger, R. L. (2002). **Statistical Inference (Second Edition).** Duxbury Advanced Series.
- [6] Rice, J. A. (2007). **Mathematical Statistics and Data Analysis (Third Edition).** Duxbury Advanced Series.

# Tentative Teaching Plan for

MA204: Mathematical Statistics

1 2 3 4 5		cture/ Date torial		Time	Content	Assignment
3	L1	Feb 13	Mon	16:20 – 18:10	§1.1 – §1.4	
3	L2				§1.5 – §1.6	
3	L3	Feb 16	Thu	08:00 - 09:50	§1.7 – §1.8	
3	<b>T1</b>				S & E	
4	L4	Feb 20	Mon	16:20 – 18:10	§1.9 – §1.10	<b>→</b> A1
4	L5				§2.1.1	
	<b>L6</b>	Feb 27	Mon	16:20 - 18:10	§2.1.2	
	<b>T2</b>				S & E	
	L7	Mar 02	Thu	08:00 - 09:50	§2.1.3 – §2.2.1	
	L8				§2.2.2 – §2.3	
5	L9	Mar 06	Mon	16:20 – 18:10	§2.4	
5	<b>T3</b>				S & E	
	L10	Mar 13	Mon	16:20 - 18:10	§2.5.1, §2.5.5, §2.6	A1  3w
	L11				§3.1.1- §3.1.3 (I)	<b>→</b> A2
	L12	Mar 16	Thu	08:00 - 09:50	§3.1.3 (II), <b>C.1</b>	
	<b>T4</b>				S & E	
6	L13	Mar 20	Mon	16:20 - 18:10	§3.1.4 – §3.2, <b>C.2</b>	
	L14				§3.3 – §3.4.1	
7	L15	Mar 27	Mon	16:20 - 18:10	§3.4.2 (I)	
	<b>T5</b>				S & E	
	L16	Mar 30	Thu	08:00 - 09:50	§3.4.2 (II)	
	L17				§3.4.3	
8	L18	Apr 03	Mon	16:20 - 18:10	§3.4.4 – §3.5	A2  3w
	<b>T6</b>				S & E	<b>→</b> A3
9	L19	Apr 10	Mon	16:20 - 18:10	§3.6 (I)	
	L20				C.3 (不考), C.4	
	L21	Apr 13	Thu	08:00 - 09:50	§4.1 – §4.2	
	<b>T7</b>				S & E	
10	L22	Apr 17	Mon	16:20 - 18:10	§4.3 – §4.5	<b>→</b> A4
	L23				§4.6 – §5.1.1	
11	L24	Apr 24	Mon	16:20 - 18:10	§5.1.2 – §5.1.3	A3  3w
	<b>T8</b>				S & E	
	L25	Apr 27	Thu	08:00 - 09:50	§5.2.1	
	L26				§5.2.2	
12		May 01	Mon	No Courses during [劳动节		
13	L27	May 08	Mon	16:20 – 18:10	§5.3	A4  3w
	T9	-: <b>-::</b>	2.2011	10.10	S & E	→A5
	L28	May 11	Thu	08:00 - 09:50	§5.4.1	
	L29	<i>j</i> 11	1110	00.00	Appendix B (不考)	

14	L30	May 15	Mon	16:20 - 18:10	Review for MT	
	<b>T10</b>				S & E	
		<b>May 20</b>	Sat	<mark>07:50</mark> - 09:50	Midterm Test	Venue: 三
					(Chapters 1-4)	教 106-107
15	L31	May 22	Mon	16:20 - 18:10	§5.4.2 – §5.4.3	
	L32				§5.5.1 – §5.5.2	
	L33	May 25	Thu	08:00 - 09:50	§5.5.3	
	<b>T11</b>				S & E	
16	L34	May 29	Mon	16:20 - 18:10	<b>Final Review</b>	
	<b>T12</b>				S & E	
		Note: J	Tune 02	(Deadline for the 5-t	<mark>ch assignment)</mark>	A5  4w
17	Exam	Jun 12	Mon	Time	19:00 - 22:00	
				Venue	三教 206, 205	

#### NOTE:

- 1. We will give a quick review on Chapter 1 because of the partial overlap of this course with "MA215 Probability" or "MA212 Probability and Statistics"
- 2. §1.11 §1.12, §3.6 (II), §4.7 will not be taught. Midterm Test and Final Examination will exclude Appendix B, Appendix C.3 and Chapter 6.
- 3. S & E = Summary & Exercise
- 4. →A1 = To assign the first assignment [布置第一次作业]
- 5. A1||3w = To submit your first assignment within 3 weeks [交第一次作业 (你有三周时间来完成此次作业)]
- 6. The course "MA204: Mathematical Statistics" has two classes:
  Class I ---- Teaching by Gary in Bilingual, with a total of 95 students enrolled;
  Class II --- Teaching by Dr. Tao LI in Bilingual, with a total of 21 students enrolled.
- 7. Midterm/Class Test of MA204 is arranged as follows:

Date: 20 May 2023 (Saturday)

**Time:** 7:50am – 9:50am (2 hours)

Venue: 三教106, 107 (Class I: 95 students), 三教107 (Class II: 21 students)

#### **Seats arrangement for the Midterm/Class Test:**

Randomly select seats.

- 8. You can prepare anything on one side of an A4 paper and bring it with you to the Midterm Test venue (期中考试允许带一张 A4 纸的单面笔记)
- 9. You can prepare anything on two sides of an A4 paper and bring it with you to the Final Examination venue (期末考试允许带一张 A4 纸的两面笔记或两张 A4 纸的单面笔记)